

FAST TRACK COMMUNICATION

Superfluid density in gapless superconductor CeCoIn₅

V G Kogan¹, R Prozorov¹ and C Petrovic²¹ Ames Laboratory and Department of Physics and Astronomy, Iowa State University, Ames, IA 50011, USA² Condensed Matter Physics, Brookhaven National Laboratory, Upton, NY 11973-5000, USAE-mail: prozorov@ameslab.gov

Received 13 January 2009

Published 13 February 2009

Online at stacks.iop.org/JPhysCM/21/102204**Abstract**

Temperature dependence of the London penetration depth λ measured in single crystals of CeCoIn₅ is interpreted as being caused by a strong pair-breaking scattering that makes the superconductivity in this compound gapless. For a gapless d-wave superconductor, we derive $\lambda = \lambda_0(1 - T^2/T_c^2)^{-1/2}$ caused by the combined effect of magnetic and non-magnetic scattering, in excellent agreement with the data in the full temperature range and with the gapless s-wave case of Abrikosov and Gor'kov. We also obtain the slope of the upper critical field at T_c that compares well with the measured slope.

(Some figures in this article are in colour only in the electronic version)

The heavy-fermion superconductor CeCoIn₅ is still under intensive scrutiny after its discovery in 2001 [1]. This is a clean (the mean free path greatly exceeds the coherence length $\xi_{ab} \approx 80$ Å), nearly two-dimensional (with small 3D pockets) [2], d-wave superconductor [3]. In the normal phase at $T > T_c = 2.3$ K, the material is a paramagnet [4]. The main interest of the community has been focused on low temperatures and high fields where the inhomogeneous Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) phase is suspected to exist. In this work we are interested in zero-field superfluid density and the c -directed upper critical field H_{c2} near T_c , the domain removed from complications of FFLO and paramagnetic constraints. Understanding the ground state properties is of utmost importance for definite predictions about the existence of more complex phases such as FFLO.

Single crystals of CeCoIn₅ were grown from an In flux by combining stoichiometric amounts of Ce and Co with excess In in an alumina crucible encapsulated under vacuum in a quartz tube [1]. The crystals used were $1 \times 1 \times 0.2$ mm³ and magnetic measurements showed practically no hysteresis.

The magnetic penetration depth was measured with a tunnel-diode resonator sensitive to changes in λ of about 1 Å. Details of the technique are described elsewhere [5]. In short, a properly biased tunnel diode maintains a self-resonating tank

circuit on its resonant frequency $\omega \sim 14$ MHz. A sample is inserted into the coil on a sapphire rod. Changes in the effective inductance cause a shift in ω proportional to the real part of the dynamic magnetic susceptibility. The system is calibrated by matching the T -dependent skin depth just above T_c . To probe λ_{ab} , a small ac magnetic field (~ 20 mOe) is applied along the c axis, so that the screening currents are flowing in the ab plane [5].

The London penetration depth $\lambda_{ab}(T)$ as a function of temperature is shown in figure 1. With excellent accuracy, the data from 0.5 K all the way to 2.1 K are described by

$$\lambda_{ab} = \lambda(0) / \sqrt{1 - t^2}, \quad t = T/T_c. \quad (1)$$

In fact, our data are close to those reported in [6] (the grey band in figure 1); our interpretation, however, is different.

Since our technique only provides information on the change $\Delta\lambda(T)$, we compare our data with those of [6], where $\lambda(0)$ was estimated from the surface impedance data. By shifting our dataset (circles in figure 1) we obtain a one-to-one correspondence with the data of [6] shown in figure 1 by the wide grey band (in fact both datasets just collapse on top of each other). The solid line is the fit to equation (1) in the full temperature interval. The only fitting parameter,

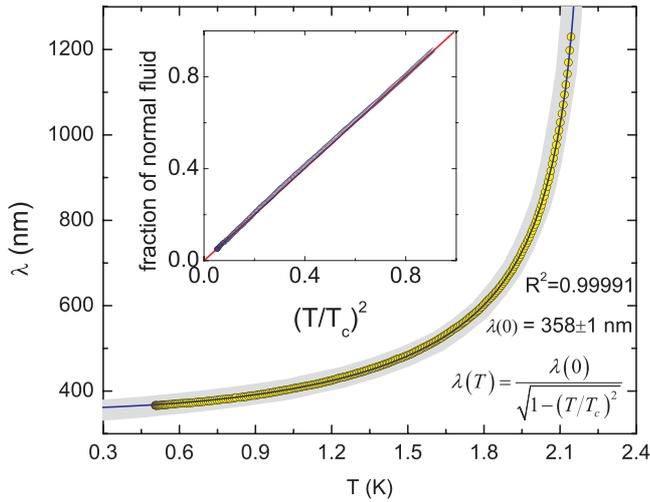


Figure 1. The data on $\lambda(T)$ and the fit to equation (1) with $\lambda(0)$ as the single fitting parameter. Grey line: data from [6]. Inset: normal fluid fraction, $1 - (\lambda(0)/\lambda(T))^2$ versus $(T/T_c)^2$ in the full temperature range. Solid line is $y = x$, not a fit.

$\lambda(0) = 358$ nm, is larger than 281 nm obtained in [6] from the low T part of the data. The fit is of a high precision, reflected in the value of the ‘coefficient of determination’ $R^2 = 0.99991$ (for a perfect fit $R^2 = 1$).

Thus, the superfluid density $1/\lambda^2$ behaves as $(1 - t^2)$ in the whole temperature domain. This is certainly not a dependence characteristic of clean d-wave materials where it should be linear at low T s (neither is it s-wave, of course). It has been argued in [7] that the linear low- T dependence might be transformed to T^2 by strong transport scattering. This, however, does not help in our case since the material of interest here is extremely clean. On the other hand, Abrikosov and Gor’kov (AG) in their seminal paper [8] on pair-breaking by magnetic impurities in isotropic s-wave materials found the dependence (1) at all temperatures from 0 to T_c for a strong spin-flip scattering when T_c is suppressed to nearly zero and the superconductivity is gapless.

This suggests that a similar situation may take place in CeCoIn_5 , although the reason for the pair-breaking may not be the spin-flip scattering on independent magnetic impurities of AG. Instead, it might be scattering on excitations of the Kondo system of interacting local moments [9]. The situation is far from being clear as is shown by transport measurements with magnetic and non-magnetic substitutions [10]. Alternatively, the gaplessness might be caused by only a part of the Fermi surface being fully gapped in multiband scenarios [11, 12].

Long experience of dealing with pair-breaking effects reveals that all of them are described by the AG formalism, provided the dimensionless pair-breaking parameters are properly defined for each particular case [13]. Below we use the AG formal scheme, having in mind, however, that the actual cause for pair-breaking is not known. Following this scheme we characterize the scattering by two parameters:

$$\rho = \hbar/2\pi T_c \tau, \quad \rho_m = \hbar/2\pi T_c \tau_m \quad (2)$$

where T_c is the critical temperature (not to be confused with the hypothetical T_{c0} of the material free of scattering), and $1/\tau$ and

$1/\tau_m$ are the transport and the pair-breaking scattering rates. As mentioned, AG find $\lambda^{-2} \propto 1 - t^2$ for sufficiently strong pair-breaking in *dirty* isotropic s-wave materials with $\rho \gg \rho_m$. The material of interest here is a clean d-wave [3]; we show below that this T dependence holds in this situation as well.

Within microscopic theory, the penetration of weak magnetic fields into superconductors is evaluated by first solving for the unperturbed zero-field state and then treating the effects of small fields as perturbations. Perhaps the simplest for this task is the Eilenberger quasiclassical approach [14]. Formally, it consists of equations for the Eilenberger functions $f(\mathbf{r}, \mathbf{v}, \omega)$, f^+ and g which originate from Gor’kov’s Green’s functions integrated over the energy near the Fermi surface to exclude fast spatial oscillations on the scale $1/k_F$:

$$\mathbf{v}\Pi f = \frac{2\Delta g}{\hbar} - 2\omega f + \frac{g}{\tau^-} \langle f \rangle - \frac{f}{\tau^+} \langle g \rangle, \quad (3)$$

$$-\mathbf{v}\Pi^* f^+ = \frac{2\Delta^* g}{\hbar} - 2\omega f^+ + \frac{g}{\tau^-} \langle f^+ \rangle - \frac{f^+}{\tau^+} \langle g \rangle, \quad (4)$$

$$1 = g^2 + f f^+. \quad (5)$$

Here, \mathbf{v} is the Fermi velocity, $\Pi = \nabla + 2\pi i \mathbf{A}/\phi_0$, ϕ_0 is the flux quantum. Δ is the order parameter which may depend on the position \mathbf{k}_F at the Fermi surface (or on \mathbf{v}) in cases other than the isotropic s-wave; for a d-wave material with a simple cylindrical Fermi surface the order parameter can be written as

$$\Delta = \Psi(\mathbf{r}, T)\Omega, \quad \Omega = \sqrt{2} \cos 2\varphi, \quad (6)$$

where φ is the azimuthal angle on the Fermi cylinder and Ω is normalized to have $\langle \Omega^2 \rangle = 1$.

Further, ω are Matsubara frequencies defined by $\hbar\omega = \pi T(2n + 1)$ with an integer n , $\langle \dots \rangle$ denote averages over the Fermi surface and

$$\frac{1}{\tau^\pm} = \frac{1}{\tau} \pm \frac{1}{\tau_m}. \quad (7)$$

The system (3)–(5) should be complemented with the self-consistency equation for the order parameter and with an expression for the current density. For the d-wave symmetry, both magnetic and non-magnetic scattering suppress the critical temperature [15]. The self-consistency equation in the form taking this into account is

$$\ln \frac{T_c}{T} = \sum_{n=0}^{\infty} \left(\frac{1}{n + 1/2 + \rho^+/2} - \frac{2\pi T}{\Psi} \langle \Omega f \rangle \right), \quad (8)$$

where $\rho^+ = \rho + \rho_m$. Finally, the current density expression completes the Eilenberger system:

$$\mathbf{j} = -4\pi |e| N(0) T \text{Im} \sum_{\omega>0} \langle \mathbf{v}g \rangle; \quad (9)$$

$N(0)$ is the total density of states at the Fermi level per one spin.

Calculation of $\lambda(T; \tau, \tau_m)$ for arbitrary τ s is difficult analytically. However, for a strong T_c suppression, the problem is manageable. We begin with a uniform zero-field state for which ρ^+ is close to the critical value where $T_c \rightarrow 0$; in this

state $f \ll 1$ and $g \approx 1 - f^2/2$ in the whole temperature range [8]. One can look for solutions of equation (3) as $f = f_1 + f_2$ with $f_2 \ll f_1$. In the lowest approximation equation (3) yields

$$2\Delta/\hbar - 2\omega f_1 + \langle f_1 \rangle / \tau^- - f_1 / \tau^+ = 0. \quad (10)$$

Since $\langle \Delta \rangle = 0$ for the d-wave materials, averaging of this equation over the Fermi surface gives $\langle f_1 \rangle = 0$ as well. Taking this into account we have

$$f_1 = \Delta / \hbar \omega^+, \quad \omega^+ = \omega + 1/2\tau^+. \quad (11)$$

The next approximation yields

$$f_2 = \frac{\Delta}{2\hbar^3\omega^{+3}} \left(\frac{\langle \Delta^2 \rangle}{2\tau^+\omega^+} - \Delta^2 \right). \quad (12)$$

Making use of $\Delta = \Omega\Psi$, $\langle \Omega^2 \rangle = 1$, $\langle \Omega^4 \rangle = 3/4$ for a Fermi cylinder, we evaluate

$$\langle \Omega(f_1 + f_2) \rangle = \frac{\Psi}{\hbar\omega^+} + \frac{\Psi^3(2 - 3\omega^+\tau^+)}{8\hbar^3\omega^{+4}\tau^+}. \quad (13)$$

Substitute this in equation (8), express the sums in terms of di-gamma functions and utilize the asymptotic expansion $\psi(z + 1/2) \approx \ln z + 1/24z^2$ for $z = \rho^+/2t \gg 1$:

$$\begin{aligned} -\ln t &= \psi\left(\frac{\rho^+}{2t} + \frac{1}{2}\right) - \psi\left(\frac{\rho^+}{2} + \frac{1}{2}\right) \\ &- \frac{3\Psi^2}{64\pi^2 T^2} \left[\psi''\left(\frac{\rho^+}{2t} + \frac{1}{2}\right) - \frac{2\rho^+}{9t} \psi'''\left(\frac{\rho^+}{2t} + \frac{1}{2}\right) \right] \\ &\approx -\ln t + \frac{t^2 - 1}{6\rho^{+2}} + \frac{\Psi^2}{48\pi^2 T_c^2 \rho^{+2}}. \end{aligned} \quad (14)$$

Hence, we obtain

$$\Psi^2 = 8\pi^2 (T_c^2 - T^2). \quad (15)$$

This differs from the result for isotropic s-wave superconductors with magnetic impurities of nearly critical concentration, by a four times larger pre-factor [8]. The ratio

$$\Delta_{\max}(0)/T_c = 4\pi \quad (16)$$

is considerably larger than 2.14 for the clean d-wave case. This ratio, estimated from the point-contact spectroscopy data for CeCoIn₅, is 12.1 ± 1.5 [16]³. Given the simplicity of the model employed (weak coupling, cylindrical Fermi surface, a source of pair-breaking still to be established) the proximity of the experimental value to the model of 4π is remarkable.

It is worth mentioning as a side remark that, even without magnetic scatterers, the superconductivity in d-wave materials becomes gapless in the domain of interest here with $\rho^+ \gg 1$. To see this, examine the density of states $N(\epsilon) = N(0) \text{Re } g(\hbar\omega \rightarrow i\epsilon)$ using $g = 1 - f^2/2 = 1 - (\Delta/\hbar\omega^+)^2/2$:

$$\frac{N(\epsilon)}{N(0)} = 1 - \frac{2\Delta^2\tau^{+2}}{\hbar^2} \frac{1 - \eta^2}{(1 + \eta^2)^2}, \quad \eta = \frac{2\tau^+\epsilon}{\hbar}. \quad (17)$$

³ Note: in fact, this paper reports two gaps for this material, $\Delta_1 = 2.4$ meV and $\Delta_2 = 0.95$ meV, so that $\Delta_1/T_c = 12.1$ whereas $\Delta_2/T_c = 4.8$.

Hence, at zero energy, $N(\epsilon)$ has a non-zero minimum (i.e. the superconductivity is gapless), whereas the maximum of $N(\epsilon)$ is reached at $\epsilon_m = \hbar\sqrt{3}/2\tau^+$.

Weak supercurrents and fields leave the order parameter modulus unchanged, but cause the condensate to acquire an overall phase $\theta(\mathbf{r})$. We therefore look for perturbed solutions in the form

$$\begin{aligned} \Delta &= \Delta_0 e^{i\theta}, & f &= (f_0 + f_1) e^{i\theta}, \\ f^+ &= (f_0 + f_1^+) e^{-i\theta}, & g &= g_0 + g_1, \end{aligned} \quad (18)$$

where the subscript 1 denotes small corrections. In the London limit, the only coordinate dependence is that of the phase θ , i.e. f_1, g_1 can be taken as \mathbf{r} -independent.

The Eilenberger equations (3)–(5) provide the corrections among which we need only g_1 :

$$g_1 = \frac{i\hbar f_0^2 \mathbf{v} \mathbf{P}}{2(\tilde{\Delta} f_0 + \hbar\tilde{\omega} g_0)} \approx \frac{i f_0^2 \mathbf{v} \mathbf{P}}{2\omega^+}. \quad (19)$$

Here $\mathbf{P} = \nabla\theta + 2\pi \mathbf{A}/\phi_0 \equiv 2\pi \mathbf{a}/\phi_0$ with the ‘gauge-invariant vector potential’ \mathbf{a} and

$$\tilde{\Delta} = \Delta + \hbar \langle f \rangle / 2\tau^-, \quad \tilde{\omega} = \omega + \langle g \rangle / 2\tau^+. \quad (20)$$

In the case of interest, $f_0 \approx \Delta/\hbar\omega^+ \ll 1$, and the denominator in equation (19) is taken in the lowest order. We now substitute $g_0 + g_1$ in the current (9) and compare the result with $4\pi j_i/c = -(\lambda^2)_{ik}^{-1} a_k$ to obtain

$$\lambda_{aa}^{-2} = \frac{32\pi e^2 N(0) v^2}{c^2 \rho^{+2}} (1 - t^2). \quad (21)$$

Using the data of figure 1 with estimates for $N(0)$ and v taken from [4], we obtain $\rho^+ \approx 8$; estimates of [3] yield $\rho^+ \approx 5$. Hence, the statement that CeCoIn₅ is gapless is not only in excellent agreement with the T dependence of λ , but the scattering parameter ρ^+ is sufficiently large for our model to hold.

There are quite a few reports of $H_{c2}(T)$ for CeCoIn₅, see, e.g., [17]. Our data are shown in figure 2. For a strong pair-breaking model with fixed scattering parameters $H_{c2} \propto (1 - t^2)$ [8], which is clearly different from the experimental behaviour. In a strong paramagnet such as CeCoIn₅, the magnetic scattering rate (and ρ_m) may itself depend on the applied field, making our model with a fixed ρ_m inapplicable *per se* along the whole $H_{c2}(T)$ curve. However, near T_c where $H_{c2} \rightarrow 0$, we expect the model to hold. We therefore evaluate only $H_{c2}(T)$ near T_c ; in other words, we derive the Ginzburg–Landau equation containing the coherence length ξ for the gapless case with a strong pair-breaking.

Near T_c , we look for a solution f of equation (3) as an expansion in two small parameters:

$$\frac{\Delta}{\hbar\omega^+} \sim \frac{\Delta}{T_c} \sim \delta t^{1/2}, \quad \text{and} \quad \frac{\mathbf{v} \mathbf{\Pi} \Delta}{\hbar\omega^{+2}} \sim \frac{\Delta \xi_0}{T_c \xi} \sim \delta t$$

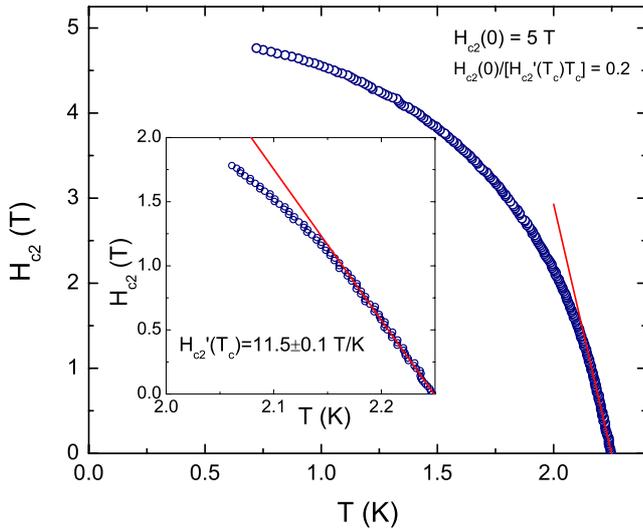


Figure 2. The upper critical field $H_{c2}(T) \parallel c$. Inset: the domain near T_c with an estimate of the slope.

where $\delta t = 1 - T/T_c$; the smallness of the second parameter comes from the ‘slow variation’ requirement. We obtain after simple algebra

$$f = \frac{\Delta}{\hbar\omega^+} - \frac{v\Pi\Delta}{2\hbar\omega^+} + \mathcal{O}(\delta t^{3/2}). \quad (22)$$

Further, the self-consistency equation (8) which is now

$$\frac{\Psi}{2\pi T_c} \delta t = \sum_{\omega>0} \left(\frac{\Psi}{\hbar\omega + \pi T\rho^+} - \langle \Omega f \rangle \right), \quad (23)$$

should be taken into account. To evaluate $\langle \Omega f \rangle$, we substitute $g = 1 - ff^+/2$ in equation (3), multiply it by Ω/ω^+ and take the average over the Fermi surface:

$$\frac{1}{2\omega^+} \langle \Omega v \Pi f \rangle = \frac{\Psi}{\hbar\omega^+} - \langle \Omega f \rangle - \frac{3\Psi|\Psi|^2}{8\hbar^3\omega^+} + \frac{\Psi|\Psi|^2}{4\hbar^3\omega^+ \tau^+}. \quad (24)$$

Writing the last two terms, we can take f in the lowest order; also we make use of $\Delta = \Psi\Omega$, $\langle \Omega^2 \rangle = 1$ and $\langle \Omega^4 \rangle = 3/4$. On the left-hand side we have

$$\begin{aligned} & \frac{1}{2\hbar\omega^+} \langle \Omega^2 v \Pi \Psi \rangle - \frac{1}{4\hbar\omega^+} \langle \Omega^2 (v \Pi)^2 \Psi \rangle \\ &= -\frac{1}{4\hbar\omega^+} \langle \Omega^2 v_i v_k \rangle \Pi_i \Pi_k \Psi; \end{aligned} \quad (25)$$

summation over the repeated indices is implied.

We now sum up equation (24) over ω and take $\sum \langle \Omega f \rangle$ from the self-consistency equation (23). All sums obtained are expressed in terms of di-gamma functions, for which the large argument asymptotics can be used. We are interested in the field along the c -crystal axis, whereas the plane ab can be taken as isotropic. After straightforward algebra one obtains the GL equation in the standard form

$$-\xi^2 \Pi^2 \Psi = \Psi(1 - |\Psi|^2/\Psi_0^2) \quad (26)$$

with

$$\xi^2 = \frac{3\hbar^2 v^2}{16\pi^2 T_c^2 \delta t}, \quad \Psi_0^2 = 16\pi^2 T_c^2 \delta t. \quad (27)$$

We note that Ψ_0^2 is the zero-field order parameter of equation (15) obtained there by a different method.

Although the scattering parameters do not enter explicitly the coherence length, they affect ξ through $T_c(\rho^+)$. Equation (27) shows that the scattering, whatever it is, *enhances* ξ . In other words, in the gapless d -wave regime, the effect of scattering upon the coherence length is opposite to the familiar s -wave situation where ξ is *suppressed* by scattering. When the scattering approaches the critical value for which $T_c \rightarrow 0$, ξ diverges. The slope of the upper critical field at $T \rightarrow T_c$ follows

$$H'_{c2}(T_c) = -\frac{8\pi\phi_0 T_c k_B^2}{3\hbar^2 v^2}, \quad (28)$$

where the temperature is given in degrees kelvin. Hence, the slope decreases when scattering is intensified. We observe that the Fermi velocity can now be expressed in terms of the measured slope and T_c . This yields $v = 0.78 \times 10^6$ cm s⁻¹, the value close to $v = 0.77 \times 10^6$ cm s⁻¹ as estimated in [3]. We note, however, that, based on the heat capacity and resistivity data, [4] reports $v = 2 \times 10^6$ cm s⁻¹.

To conclude, the data on the T dependence of the penetration depth in the *full temperature range* and on the upper critical field near T_c strongly support the notion that the superconductivity in CeCoIn₅ is well described by the presence of a strong pair-breaking which makes the superconductivity in this compound gapless. A large experimental ratio Δ_m/T_c for this compound also follows from the combination of the d -wave symmetry and a strong pair-breaking, equation (16).

We thank P Canfield, S Bud'ko and M Tanatar for many useful discussions. The work at the Ames Laboratory is supported by the Office of Basic Energy Sciences of the US Department of Energy under contract no. DE-AC02-07CH11358. RP acknowledges the support of the Alfred P Sloan Foundation. Part of CP's work was carried out at the Brookhaven National Laboratory operated for the US DOE by Brookhaven Science Associates (DE-Ac02-98CH10886).

References

- [1] Petrovic C *et al* 2001 *J. Phys.: Condens. Matter* **13** L337
- [2] Settai R *et al* 2001 *J. Phys.: Condens. Matter* **13** L627
Settai R, Takeuchi T and Onuki Y 2007 *J. Phys. Soc. Japan* **76** 051003
- [3] Movshovich R *et al* 2001 *Phys. Rev. Lett.* **86** 5152
Izawa K *et al* 2001 *Phys. Rev. Lett.* **87** 057002
- [4] Petrovic C *et al* 2002 *Phys. Rev. B* **66** 054534
- [5] Prozorov R *et al* 2000 *Phys. Rev. B* **62** 115
Prozorov R and Giannetta R W 2006 *Supercond. Sci. Technol.* **19** R41 (topical review)
- [6] Ozcan S *et al* 2003 *Europhys. Lett.* **62** 412
- [7] Hirschfeld P J and Goldenfeld N 1993 *Phys. Rev. B* **48** 4219
- [8] Abrikosov A A and Gor'kov L P 1960 *Zh. Eksp. Teor. Fiz.* **39** 1781

- Abrikosov A A and Gor'kov L P 1961 *Sov. Phys.—JETP* **12** 1243 (Engl. Transl.)
- [9] Nakatsuji S, Pines D and Fisk Z 2004 *Phys. Rev. Lett.* **92** 016401
- [10] Paglione J *et al* 2007 *Nat. Phys.* **3** 703
- [11] Tanatar M A *et al* 2005 *Phys. Rev. Lett.* **95** 067002
- [12] Barzykin V and Gor'kov L P 2007 *Phys. Rev. B* **76** 014509
- [13] Maki K 1969 *Superconductivity* vol 2, ed R D Parks (New York: Dekker) p 1035
- [14] Eilenberger G 1968 *Z. Phys.* **214** 195
- [15] Openov L A 1997 *JETP Lett.* **66** 661
- [16] Rourke P M C, Tanatar M A, Turel C S, Berdelis J, Petrovic C and Wei J Y T 2005 *Phys. Rev. Lett.* **94** 107005
- [17] Bianchi A *et al* 2003 *Phys. Rev. Lett.* **91** 187004